Mathematical Monism: Pythagoras, Plato, Wheeler, Zuse, Fredkin, Geroch, Balaguer, Ladyman, French, Marchal, Schmidhuber, Tegmark

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This paper advances a strict thesis: the only reality we can justifiably claim to know is mathematics. We frame this claim historically and conceptually through a lineage of thinkers who, in different idioms, converge on mathematical monism. From Pythagoras's "all is number" to Plato's elevation of mathematical knowability over sensible flux, the ancient seed is clear: truth is mathematical order. In the twentieth century, Wheeler's "it from bit" and Zuse's *Rechnender Raum* pivot the thesis toward information and computation as the ontic fabric, while Fredkin's digital physics sharpens the identity between physical and informational structure. Geroch's austere stance in mathematical physics compresses theory and world into structure alone; Balaguer's Full-Blooded Platonism universalizes existence to all consistent mathematical objects. Ladyman and French's Ontic Structural Realism argues that relations—not objects—are fundamental, aligning ontology with mathematics as pure structure. Marchal's arithmetical realism roots observers and physics inside arithmetic itself; Schmidhuber's Algorithmic TOE enumerates all computable universes; Tegmark's MUH asserts unambiguously that our universe is a mathematical structure. By placing these voices in historical order, we map a tightening spiral from metaphor to identity, arguing that "math-only" is not poetry but the sober terminus of explanation.

Keywords: mathematical monism, Pythagoras, Plato, Wheeler, Zuse, Fredkin, Geroch, Balaguer, Ladyman, French, Marchal, Schmidhuber, Tegmark, digital physics, ontic structural realism, arithmetical realism, algorithmic universe, mathematical universe hypothesis, full-blooded platonism, computable universes, information monism, cellular automata.

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I. Introduction: Why "only mathematics"

We begin by stating the thesis in its strictest form:

Mathematical Monism (MM): Reality = mathematics. There are no mind-independent entities over and above mathematical structures; what we call "physical" is an internal perspective on mathematics.

This is stronger than (i) mathematical descriptivism ("math is our best language for physics"), stronger than (ii) epistemic monism ("only mathematical objects can be known with certainty"), and stronger than (iii) structural realism ("only structure is real, but it is instantiated in a physical domain"). MM asserts *identity*, not mere isomorphism.

Why prefer identity over isomorphism? If every admissible empirical regularity is captured by some mathematical structure S, and if all counterfactuals licensed by our best theories are exactly those of S, then positing an extra "substance" that instantiates S adds no testable content. Occam's razor says: collapse the map and the territory—there is just S. Formally:

- Let T be a successful theory whose models are structures M(T).
- Let E be total empirical evidence across accessible interventions.
- If for some S in M(T), all predictions/retrodictions/counterfactuals derivable from S match E to within error bounds, and no additional non-mathematical posit yields novel, discriminating predictions, then the non-mathematical posit is explanatorily idle.
- Replace "is instantiated by S" with "is S."

Explanatory advantages.

1. **Unification:** MM removes the dual bookkeeping of "laws vs. stuff," replacing it with a single ontology of structure. Conservation, symmetries, and dynamics become properties of the structure, not rules imposed upon a substrate.
2. **Modal clarity:** Possibility = consistency relative to axioms. The space of "physically possible" worlds becomes the space of mathematically consistent structures satisfying appropriate constraints. This offers a clean semantics for counterfactuals used in physics.
3. **No-miracles argument (recast):** The unreasonable effectiveness of mathematics is no miracle if the world *is* mathematics. Success of advanced mathematics in novel domains (e.g., gauge theory, renormalization, symmetry breaking) reads as discovery, not coincidence.
4. **Algorithmic economy:** If the shortest description of our universe is a mathematical program/model P, then any "non-mathematical" remainder increases Kolmogorov

complexity without increasing predictive power. Prefer minimal description length: world = P.

Scope and falsifiability. Critics ask whether MM is unfalsifiable. The right standard is *comparative* adequacy. MM is disfavored only if a rival ontology (math + extra substrate) yields strictly stronger, validated predictions or strictly lower description length for the same predictive power. Otherwise, MM wins on parsimony and invariance.

Identity vs. representation. A recurring confusion: "But equations live on paper; rocks do not." MM distinguishes *tokens* from *types*. Written symbols are tokens used by agents within a substructure; the reality claimed by MM is the abstract structure those tokens denote. The rock is not ink; the rock is a node inside the total structure whose intrinsic properties are exactly its structural profile.

The measurement/experience objection. "What about qualia and measurement?" Under MM, observers are definable substructures with decision procedures, memory states, and update rules. Measurement is an interaction pattern among substructures satisfying information-theoretic constraints. Experience, if posited, supervenes on (and is identical to) specific high-level structural invariants. The explanatory burden is to *exhibit* those invariants, not to add non-mathematical essence.

Underdetermination. Could multiple inequivalent mathematical structures fit the data? Yes; science already resolves this via added constraints: symmetry principles, locality, computability bounds, renormalizability, and information-theoretic postulates. MM does not amplify underdetermination—it inherits it and then disciplines it by preferring structures with maximal invariance and minimal description length compatible with data.

Working principle (operational MM):

- Postulate: world = the simplest structure S* consistent with current evidence and meta-constraints (symmetry, locality, computability).
- Progress criterion: refine S* by tightening axioms and compressing description length without loss of empirical reach.

The remainder of the paper shows that, across history, arguments for "math as description" repeatedly sharpen into "math as identity," culminating in modern programs (digital physics, full-blooded Platonism, algorithmic universes, MUH) that operationalize this identity claim.

II. Pythagoras: All is number, ratio, harmony

Thesis. In the earliest rigorous form, mathematical monism appears as the Pythagorean identification of being with number: *ta onta arithmoi eisin*—"things are numbers." Not "things are *described by* numbers," but that what makes a thing what it is is a numerical structure of relations (ratio, proportion, harmony). Ontology = arithmetic/ratio.

From sound to substance. The canonical experiment—fix a lyre string and vary its length or tension—yields stable consonances at simple integer ratios (1:2 octave, 2:3 fifth, 3:4 fourth). For Pythagoreans this is not contingent acoustics but a disclosure of reality's grammar: simplicity of **ratio** grounds stability of **form**. Harmony (harmonia) is thus a structural invariant; the world's order is a scale whose intervals are ratios.

Peras and apeiron (limit and unlimited). Pythagorean metaphysics pairs the *limited* (peras) with the *unlimited* (apeiron). Number, especially **integer ratio**, is the power that imposes limit on the indeterminate. Being is formed by the act of ratioing—rendering the indefinite commensurable. Cosmology follows: cosmos (order) is literally a measured fitting-together.

The tetraktys as ontology, not symbol. The sacred tetraktys (1+2+3+4 = 10) encodes the generative scaffold: unity, duality, triadic relation, and tetradic closure. Geometry, music, arithmetic, and cosmology are "unfoldings" of this seed. Oath to the tetraktys is not numerology; it is a vow to a structural source of being.

Two schools, one monism.

- *Akousmatikoi* (listeners): ethical-ascetic program—purify the soul by tuning it to numerical order.
- *Mathematikoi* (learners): constructive proofs in arithmetic/geometry and the founding of proportion theory. Both treat number as **constitutive** of reality: ethics becomes tuning-to-structure; science becomes unveiling-structure.

Philolaus's fragments (proto-axioms). Philolaus articulates that "nature in the cosmos is fitted together out of the unlimited and the limiting," with **number** the principle of fitting. Objects are not primitive substances; they are **commensurated patterns**—a striking anticipation of "objectless" structure.

The scandal of the incommensurables. Discovery of √2 (and later irrationals) is portrayed as a crisis: not all magnitudes share a common unit. Far from refuting monism, it **refines** it: reality is not exhausted by integer ratios, so the ontology of number must widen from discrete arithmetic to a general theory of proportion and magnitude (eventually Eudoxus and, later, real analysis). The lesson for strict monism: when the world resists a *representation class* (integers), upgrade the **structure class** (ratios → proportions → reals) rather than abandoning identity.

Ethics and politics as applied mathematics. The "well-tempered soul" and well-ordered polis mirror the well-tuned scale: justice is proportion, virtue is mean, friendship is ratioed reciprocity. Monism here is lived: to be good is to be in tune with number.

Continuities to modern programs.

- **Structural primacy:** properties supervene on relations—ancestral to structural realism.
- **Compression ideal:** consonance = low-description-length relation—anticipating MDL/algorithmic parsimony.
- **Modal insight:** the possible is the proportionable—foreshadowing "possibility = consistency."

Why Pythagoras matters for Mathematical Monism. He supplies the **template**: (i) identify a domain where structure is transparently real (music/geometry), (ii) elevate that structure to universal ontology, (iii) resolve anomalies by **expanding mathematics**, not by re-importing non-mathematical "stuff." The Pythagorean move converts "math works" into "math *is*," launching the arc that later reaches digital physics, algorithmic universes, and MUH.

III. Plato: *Epistēmē* and mathematical knowability over *sensibles*

Claim in this lineage. Plato does not say "reality = mathematics" in the strict modern sense, yet he makes the stronger epistemic thesis that **only what is mathematically/formally structured is knowable** (*epistēmē*), whereas the flux of sensibles yields at best belief (*doxa*). If our paper's standard is "the only reality we can *justify as known* is mathematics," then Plato is a principal ancestor.

1) The Divided Line: a graded ontology of knowability.
In *Republic* VI, the famous line orders cognition and being:

- **A)** *Eikasia* (images, shadows) → weakest grasp, least reality.
- **B)** *Pistis* (ordinary objects) → belief about changeable things.
- **C)** *Dianoia* (mathematics) → reasoning with hypotheses, diagrams, and proofs about invariant relations.
- **D)** *Noēsis* (dialectic) → direct intellection of Forms, culminating in the Good.

Crucially, levels C–D are **non-sensible** and regimented by necessity; they are accessed by proofs and definitions. Mathematical thinking (C) is the *gateway* to noetic knowledge (D), and its

object—structure—is more real than sensibles. Thus, what we can *know* with necessity is mathematical in kind.

2) Mathematics as training for truth.
Plato's curricular "ladder" (*Republic* VII) moves from arithmetic to plane/solid geometry, astronomy, and harmonics—not as applied calculation, but as **purification of the intellect**. Mathematics imposes *peras* (limit, exactness) on the *apeiron* (the indefinite), echoing the Pythagorean "fitting together" of reality by ratio. The point is not utility; it is reorientation from becoming to being, from contingency to invariance.

3) The *Timaeus*: a crafted cosmos by measure.
The Demiurge orders pre-cosmic disorder by **number, proportion, and geometric solids**, sewing soul and body of the cosmos through harmonic ratios and the Platonic solids. Even if mythic in tone, the ontology is clear: *cosmos = mathematized order*. The physical is intelligible *insofar as* it has been numerically proportioned. What counts as genuinely explanatory is mathematical fitting, not material push-and-pull.

4) The ontological-epistemic hinge.
Plato famously distinguishes **Forms** from **mathematicals** (intermediates). Yet two facts matter for our strict thesis:

- The **criterion of knowledge** is **necessity**; sensibles cannot meet it, mathematics can.
- The **method of ascent** to the highest principle (the Good) proceeds **through** mathematics; dialectic "removes" hypotheses but preserves structural necessity.

Hence, if we confine ourselves to what can be *known*, mathematics provides the **full positive content**. The "more-than-mathematical" Form of the Good grounds this order, but does not add empirical content beyond structure. Epistemically, mathematics is the **entire deliverable**.

5) Objections and replies.

- **Objection (Intermediates):** If mathematicals are "between" Forms and sensibles, mathematics is not ultimate.
  Reply: Our thesis is **epistemic**: what we can *know* with certainty is exhausted by structural relations. Whether Forms outrank mathematicals metaphysically is orthogonal to the question of knowable content.
- **Objection (Third Man / anti-reification):** Pure Forms outrun mathematical description.
  Reply: Even if the Good "exceeds" being, Plato's own pedagogy makes **mathematical necessity** the measure of intellectual progress; it is the *currency* of justification. The excess does not yield additional, testable ontology.

- **Objection (Physics of flux):** Empirical science studies sensibles successfully.
  Reply: For Plato, such success occurs **insofar as** sensibles are brought under mathematical regularity (astronomy done right is geometry of motions, not stargazing). Where flux resists mathematization, it resists knowledge.

6) Measure, limit, and mixture (*Philebus*).
The quartet—*limit* (peras), *unlimited* (apeiron), *mixture*, and *cause*—assigns **measure** and **ratio** the role of making anything *knowable*. Qualities become tractable when commensurated; value (the good life) is likewise a matter of proportion. The ethical upshot mirrors the epistemic: truth and goodness travel with measure, i.e., with mathematics.

7) Plato's place in mathematical monism.

- **Not**: "reality = mathematics" (ontological identity).
- **But**: "only mathematics (and its higher structural kin) is **knowable**."
  Combine that with the *Republic–Timaeus* program—where explanation cashes out in number—and Plato functions as **epistemic monist**: the *entire* content of justified knowledge is mathematical/formal structure.

Working theses for this section.

1. *Epistemic Priority*: Mathematics supplies the only necessary, and thus knowable, content.
2. *Explanatory Sieve*: Explanations that survive dialectic are structural; non-structural posits are idle.
3. *Continuity Claim*: Modern "math-only" programs (structural realism, digital physics, MUH) inherit Plato's filter: whatever is **not** mathematically capturable does **not** count as knowledge.

In our arc, Plato thus fixes the rule that later monists radicalize: **where there is knowledge, it is mathematics; where there is no mathematics, there is no knowledge.**

IV. Wheeler: It-from-bit and informational primacy

Claim in this lineage. Wheeler reframes ontology in terms of information: physical reality (the *it*) arises from discrete yes/no distinctions (the *bit*). The world is not fundamentally made of fields or particles but of answers to binary questions. If information is formal, and formal = mathematical, then Wheeler's move is a bridge from physics to mathematical monism: reality is the calculus of distinctions.

Core ideas.

1. **Yes/No as primitive.** At the deepest level, the universe is constituted by elementary propositions with binary outcomes. A measurement records a bit; the accumulation of such records is the world's "facticity."
2. **Participatory universe.** Observers are not external auditors; the posing of questions helps bring the phenomena into being. Observation does not merely reveal a value; it contributes the bit that concretizes it.
3. **Law without law.** Regularities need not be imposed by prior substance-like "laws." Stable patterns can emerge from informational constraints and selection across histories. Law is a statistical invariant of information processes, not a decree etched into matter.
4. **Delayed-choice and quantum inference.** Experiments suggest that what counts as a "fact" depends on the information network into which outcomes are embedded. Causality, in everyday terms, is an emergent bookkeeping over bits, not an underlying push-pull mechanism.

From information to mathematics. To use Wheeler for strict mathematical monism, we need the identification step:

- **Information is formal.** A bit is a 2-element partition; a record is a finite string over {0,1}; an experiment is a decision procedure mapping contexts to bit strings. These are mathematical objects (sets, strings, algorithms, sigma-algebras).
- **Constraints = theorems.** Conservation, symmetry, and no-signaling are statements about allowable mappings among strings and measures over outcome spaces. Replace "physical law" with "admissible transformations of information."
- **Compression principle.** If the shortest description of an observed bit history is a theory T (a program or axiomatic structure) of length L(T), then any extra "substance" that does not reduce L(T) is idle. Information theory and MDL point to mathematics as the minimal ontology for what is actual.

Hence the Wheeler pipeline:
bits -> codes/measures -> invariants under transformations -> mathematics as the unique bearer of these invariants.

Objections and replies.

- **Objection (Info needs a medium).** Bits require a physical substrate; therefore information cannot be fundamental.

Reply. Substrate talk reappears only when we reify implementation details. In MM terms, "substrate" is a substructure that implements the code. The identity is with the code-theoretic structure, not a separate stuff.

- **Objection (Information vs. meaning).** Bits are syntactic; physics needs semantics.
  Reply. The semantics in physics is given by invariants and calculi relating questions and answers (e.g., Born rule constraints). These are mathematical. What remains "semantic" beyond this does not change predictions and thus is explanatorily idle.
- **Objection (Observer dependence).** Participatory accounts risk anthropocentrism.
  Reply. Observers in Wheeler's sense are generic interrogators: any substructure capable of registering distinctions. Their role is definable in computational/information-theoretic terms; no human exceptionalism is required.

Operational program (Wheeler-to-MM).

1. **Primitive:** Questions = sigma-algebra on an outcome space; answers = bits.
2. **Dynamics:** Valid update rules are stochastic maps preserving constraints (no-signaling, unitarity-like invariants).
3. **Emergence:** Macroscopic "objects" = stable codes with low Kolmogorov complexity under coarse-graining.
4. **Counterfactuals:** Modal space = set of all bit histories consistent with constraints; physics = measure over this set.
5. **Criterion:** Prefer theories that minimize description length while maximizing invariance (Occam + symmetry).

Relation to other sections. Wheeler tightens Plato's epistemic filter (knowable = formal) and Pythagoras's ratio ontology (structure first) by relocating the locus of being to recorded distinctions. He becomes the modern hinge: once "it" is recast as "bit," the step to "math-only" is no longer rhetorical but procedural. If everything that exists is a network of well-posed questions and constraint-satisfying answers, then everything that exists just is the mathematical structure that encodes those questions, answers, and constraints.

V. Zuse: *Rechnender Raum* and cellular-automaton ontology

Thesis in this lineage. In *Rechnender Raum* (1969), Konrad Zuse proposed that physical reality is a discrete computation: space itself is a vast cellular automaton (CA). The universe does not merely *obey* equations; it *executes* an update rule on a lattice of elementary cells. This is an

ontological claim: what exists are cells, states, and local transition functions. If a CA is a purely mathematical object (alphabet, grid, neighborhood, rule), then Zuse's cosmos is mathematics running on mathematics.

Core construction.

- **Alphabet:** a finite set of cell states, e.g., {0,1,...,k-1}.
- **Lattice:** typically a regular grid (Zuse envisioned a 3D array), possibly time-sliced as layers t = 0,1,2,...
- **Neighborhood:** a finite stencil around each cell (e.g., von Neumann or Moore neighborhood).
- **Local rule:** f: Sigma^N -> Sigma mapping neighborhood configurations to a next state.
- **Global evolution:** applying f synchronously yields a deterministic map U: Sigma^(L) -> Sigma^(L).

Every element here is mathematical: sets, functions, and synchronous iteration. The "physical" becomes the orbit of the global map U from an initial condition x_0. Motion, interaction, and fields are reinterpreted as patterns propagating through U's causal cone.

Why a CA matches physical desiderata.

1. **Locality:** Next states depend on finite neighborhoods, mirroring finite signal speed.
2. **Causality:** The global evolution is an explicit time-ordered map; past light-cones are sublattices of influence.
3. **Conservation and symmetry (by design):** Choose f to respect discrete analogs of conservation or symmetry constraints; global invariants become theorems about U.
4. **Computability:** The universe's evolution is a computation; prediction reduces to running U, aligning ontology with algorithm.

From representation to identity. A common soft view says "a CA can simulate physics." Zuse's stronger claim is **identity**: the world *is* the CA. Under mathematical monism, we erase any supposed difference between a "physically real CA" and the CA as a structure. There is just the structure (L, Sigma, f, x_0) and its orbit.

Continuities and upgrades.

- **Pythagoras -> Zuse:** ratio/harmony becomes local rule/invariant; order is generated by repeated application of f.

- **Plato -> Zuse:** knowable content is structural; Zuse operationalizes it with an explicit update rule.
- **Wheeler -> Zuse:** “it from bit” receives a concrete substrate: bits on a lattice, updated by f.

Emergence of objects and laws.

- **Particles as patterns:** Localized, persistent configurations (gliders, oscillators) function as “particles.”
- **Interactions as collisions:** When patterns’ light-cones overlap, the joint update yields scattering, bound states, or annihilation.
- **Fields as codes:** Slowly varying, extended configurations can serve as discrete field analogs.
- **Laws as invariants:** Conservation and symmetries are statements about U’s preserved quantities or automorphism group.

Algorithmic virtues.

- **Finite description length:** A small rule table plus lattice geometry compresses vast regularities; MDL favors such theories.
- **Decidability profile:** Some questions become decidable/semidecidable relative to U; others inherit CA-level undecidability, explaining limits of prediction without adding metaphysical opacity.
- **Testability by compilation:** To confront data, we compile observed regularities into candidate f and check whether U reproduces phenomena across scales under coarse-graining.

Objections and replies.

- **Continuity objection:** Physics appears continuous and Lorentz invariant; CAs are discrete and lattice-bound.
  Reply: (i) Effective continuum limits via renormalization/coarse-graining can recover differential behavior; (ii) discrete models can approximate Lorentz symmetry statistically or in emergent regimes; (iii) MM’s standard is comparative adequacy and description length, not exact preservation of pretheoretic form.
- **Synchrony objection:** Global synchronous updating seems unphysical.
  Reply: Asynchronous or causal CAs (partial orders, update by local clocks) exist; the essential point is local rulehood, not strict synchrony.

- **Parameter arbitrariness:** Which lattice, alphabet, and rule?
  Reply: This is the scientific task: infer the simplest U that compresses data and maximizes invariance. Zuse's contribution is the ontological posture and constructive template.

Legacy and cross-links.

- **Fredkin and digital physics:** Recasts Zuse's vision with reversible CAs and information as the concrete "stuff."
- **Schmidhuber's algorithmic universes:** Elevates the Zuse-style construction to an ensemble of all programs, positioning our world as one computable orbit.
- **Wolfram/Rule-based physics:** Explores rich phenomenology and emergent complexity from simple rules, reinforcing the plausibility of CA ontologies.

Why Zuse matters for Mathematical Monism. Zuse supplies the first fully explicit **identity model** in which what exists is nothing over and above a mathematical automaton. No dualism of law and substrate is required: the "law" is the rule f; the "substrate" is the state space Sigma^(L); "becoming" is iteration by U. If the universe can be captured by so lean a structure without loss of predictive reach, then, by parsimony and invariance, mathematics is not merely our language for the world; it is the world.

VI. Fredkin: Digital physics and identity of information with the real

Thesis in this lineage. Edward Fredkin pushes the Pythagoras→Wheeler→Zuse arc to an ontological punchline: **information is the real**. Not merely "represented by" the real; **identical with** it. The cosmos is a computation whose states and updates are informational through-and-through. If the world is nothing over and above the evolution of information, and if information is formal/mathematical, then mathematical monism follows by identity, not analogy.

Key pillars.

1. **Reversibility as first principle.**
   Fredkin insists on *reversible* dynamics: every state has a unique predecessor. In logic-circuit terms, the dynamics is a bijection on the global state space. This has two consequences:
   - **Conservation:** Reversibility ensures no loss of "information mass." (In conservative logic, logical operations preserve a quantity analogous to energy.)

- **Time symmetry:** If the microdynamics is one-to-one, there's no fundamental arrow of time baked into the ontology; entropy growth is epistemic/coarse-grained, not ontic.

2. **The Fredkin gate and conservative logic.**
   The 3-input Fredkin gate (a controlled swap) is reversible and functionally universal for classical computation while preserving "particle counts" under certain encodings. This yields a **physics-like** computation: signals can be treated as conserved entities whose interactions implement logic. The point is not gadgetry; it is an **existence proof** that rich, lawlike behavior emerges from strictly information-preserving rules.

3. **Digital physics program.**

   - **Finite symbolic alphabet** (bits or small integers).
   - **Locally interacting, reversible update** (e.g., reversible cellular automata or billiard-ball models à la Toffoli–Margolus).
   - **Macro-laws as invariants and statistics of micro-updates.** Fields, particles, and forces become codings of persistent, mobile patterns; conservation laws become theorems about the permutation structure of the global update.

4. **Identity thesis clarified.**
   "Information is the most concrete thing in the universe" is not epistemic bravado; it is an **ontological identification**: the world *is* the evolving pattern of distinctions, with no extra "stuff" underneath. Apparent material properties supervene on (and are identical to) information-theoretic invariants: locality, reversibility, causal cones, and symmetries of the update.

From digital physics to mathematical monism.

- A reversible computation is a bijection $U: \Sigma \to \Sigma$on a discrete state space Σ.
- A history is an orbit $\{U^t(x_0)\}_{t\in\mathbb{Z}}$.
- All objects/events are definable properties of that orbit (e.g., predicates on finite windows).
  Everything here—Σ, U, orbits, predicates—is **mathematical**. Postulating a distinct "physical substrate" adds no predictive content beyond the structure $\langle \Sigma, U, x_0 \rangle$. By Occam and MDL, discard it: **world = structure**.

Explanatory payoffs.

- **Arrow of time demystified.** Microscopic reversibility with macroscopic entropy growth follows from typicality and coarse-graining over high-dimensional Σ. No mysterious "flow" is needed—just counting arguments on orbits.
- **Conservation laws as group facts.** If U commutes with a group action G on Σ, the corresponding quantity is conserved. "Energy" and "momentum" translate to invariants under time-translation and lattice symmetries; these are theorems about automorphisms, not extra substances.
- **Particles and interactions as codewords and collisions.** Stable, localized configurations (codewords) propagate; interactions are compositions of local bijections. Scattering amplitudes become statistics of collision types—again, structural facts.

Operational program (Fredkin-to-MM).

1. **Pick the reversible substrate:** a reversible CA or lattice gas with local, bijective rules.
2. **Impose symmetry desiderata:** translation invariance, isotropy (as feasible), and conservation-like constraints.
3. **Search for compression:** identify minimal rule tables that compress observed regularities (MDL).
4. **Derive macros:** coarse-grain to effective field equations; validate across scales.
5. **Retire surplus metaphysics:** wherever a "material" posit does not change predictive reach or reduce description length, eliminate it.

Common objections and replies.

- **"But quantum!"** Reversible classical dynamics seems too weak.
  Reply: Reversibility, locality, and information primacy are compatible with quantum generalizations (unitarity as reversible linear dynamics; quantum cellular automata; finite-information constraints). Fredkin's core thesis targets *ontology* (information/structure), not a specific classical formalism.
- **"Discrete lattices violate Lorentz symmetry."**
  Reply: Exact symmetry may be replaced by emergent/approximate symmetry in continuum limits; the metric itself can emerge from causal/informational order. The metric-free causal set and quantum walk literatures show routes from discrete to relativistic behavior.

- **“Information requires a medium.”**
  Reply: The “medium” is the state space $\Sigma$and its update U—already the mathematical object. A further “stuff” bearing the structure adds no testable content.

Why Fredkin matters for mathematical monism.
He supplies the **identity bridge**: once physics is recast as strictly reversible information processing, all explanatory work is done by mathematical structure—state spaces, permutations, symmetries, and statistics. Nothing remains for non-mathematical ontology to do. In Fredkin’s universe, to *be* is to be an element of Σon an orbit under U; to *act* is to participate in a local bijection; to *explain* is to prove a structural invariant. That is mathematical monism in operational form.

VII. Geroch: Physics as mathematics; structure sans surplus ontology

Thesis in this lineage.
Geroch’s program in mathematical physics treats a physical theory as **nothing but a class of mathematical structures** together with a notion of equivalence among them. What we typically call “physical facts” are **invariants of that structure**. When two models are related by a symmetry (e.g., an isometry), they represent the *same* physical situation. This stance removes all “surplus ontology” beyond structure and pushes the identity move: *physics = mathematics up to the theory’s equivalences*.

Core commitments.

1. **Model-first formulation.**
   A theory $T$is given by a space (or category) of models $\mathcal{M}(T)$: e.g., in GR, oriented, time-oriented Lorentzian manifolds (M, g_{ab})satisfying Einstein’s equations, possibly bundled with matter fields (sections of vector bundles) that satisfy their field equations.

2. **Physical sameness = mathematical equivalence.**
   The physically relevant content is the **equivalence class** $[M, g, \dots]$under the theory’s symmetries (for GR, diffeomorphisms that carry solutions to solutions). Observables are **diffeomorphism-invariant functionals** on $\mathcal{M}(T)$; “points of spacetime” have no independent identity apart from the fields.

3. **Eliminate primitive “substances.”**
   Manifold points, coordinates, and background structures are bookkeeping devices. If a posit cannot be expressed as an invariant of the model (mod symmetries), it is not part of the world according to the theory.

4. **Algebraic reformulations.**
   The same content can be recast without points altogether (e.g., Einstein-algebra style): replace $(M, g)$by suitable commutative algebras with additional structure encoding the metric and derivations. What matters is the **structure**, not its presentation. Equivalent mathematical presentations = the same physics.

Formal picture (GR as exemplar).

- **Objects:** Models $(M, g_{ab}, \psi)$where $g_{ab}$is a Lorentzian metric and ψdenotes matter fields, all satisfying a coupled system of PDEs.
- **Morphisms:** Smooth maps $\varphi: M \to M'$that pull back solutions to solutions (isometries with appropriate pullbacks on bundles).
- **Invariants:** Geometric/scalar concomitants of g, ψ(curvature scalars, conserved currents), global/topological data (causal structure, Cauchy surfaces, holonomy).
- **Dynamics:** Initial-value formulation: data on a Cauchy 3-surface Σsatisfying constraints evolve (uniquely up to diffeomorphism) to a spacetime. "Uniquely up to diffeo" is the tell: *only the equivalence class* is determined.

This architecture dissolves the "hole argument" type worries: if diffeomorphic models are physically the same, there is no underdetermination; there is only **representational redundancy**.

From structure-only physics to mathematical monism.

- Step 1: All empirical content is encoded as invariants of models in $\mathcal{M}(T)$.
- Step 2: Two distinct *presentations* that are isomorphic represent one physical situation.
- Step 3: Posits that are not invariant (coordinates, labels, manifold points without fields) are surplus.
- Step 4: Therefore, the "physical situation" **just is** the mathematical isomorphism class $[X]$of a model X. No additional non-mathematical "stuff" remains to be identified.

Explanatory dividends.

- **Parsimony:** Laws become constraints on admissible structures; explanations are proofs about invariants, not stories about substrata.
- **Modal discipline:** Possibility becomes membership in $\mathcal{M}(T)$; counterfactuals are comparisons within (or between) model classes.

- **Presentation-independence:** Multiple equivalent formalisms (tensor, spinor, algebraic, categorical) are *not* different ontologies; they are **functorially** related views of the same content.
- **Local–global coherence:** Theorems about causal structure (global hyperbolicity, singularity/extension results, splitting theorems) are literal statements of what the world permits if the world *is* such a structure.

Common objections and replies.

- **"But where is matter?"**
  As sections of bundles with their own field equations and stress-energy; all encoded structurally. If two models agree on all invariant matter quantities, there is no further question of "which stuff."
- **"Mathematics describes; it doesn't *exist*."**
  The Geroch move is precisely to collapse "describes" into "is," once description is **complete up to equivalence** and no further empirically discriminating posit is available. Adding "stuff" that leaves $[X]$unchanged adds no empirical content.
- **"Different theories fit the same data (underdetermination)."**
  True in general; but this is a feature of science, not a defect of structuralism. The comparative standard remains: pick the structure class with maximal invariance, empirical reach, and minimal description length (MDL). When a rival class yields strictly finer, validated predictions, we trade up.
- **"Where do observers and measurements live?"**
  As definable substructures (worldlines, detectors as localized field configurations, measurement interactions as couplings). Outcomes are invariants (e.g., transition probabilities, scattering data) of the full model.

Operational program (Geroch-style).

1. **Specify the model space:** define precisely the objects and morphisms; state symmetry principles as categorical equivalences.
2. **State the dynamics as constraints:** PDEs/variational principles selecting $\mathcal{M}(T) \subset$all candidate structures.
3. **Extract observables as invariants:** quantities functorial under morphisms; banish gauge/coordinate artifacts.
4. **Compare theories by invariants & MDL:** prefer the theory whose invariant content compresses the data best and whose symmetry structure is richest and cleanest.

5. **Translate across presentations:** maintain dictionaries (tensor ↔ spinor ↔ algebraic) showing presentation-independence.

Why Geroch matters for mathematical monism.
Geroch supplies the **exact forcing step** from "math describes" to "math is": if physics is exhausted by the isomorphism class of a model and all empirical content is invariant under the theory's symmetries, then there is **no remainder** for a non-mathematical ontology to explain. The world, on this view, is a particular equivalence class $[X]$inside a rigorously defined mathematical category of models. Explanations are proofs about $[X]$; predictions are theorems about how invariants evolve under the dynamics; observation is identification of invariant features of $[X]$. That is physics **as** mathematics—structure without surplus.

VIII. Balaguer: Full-Blooded Platonism and universal mathematical existence

Thesis in this lineage.
Mark Balaguer's Full-Blooded Platonism (FBP) claims: *every* consistent mathematical structure exists. No privileged domain, no ontic scarcity: if a structure is logically coherent, it is real in the same robust sense as the reals, groups, manifolds, large cardinals, etc. FBP thus supplies the **maximal existence principle** for mathematics.

Core commitments.

1. **Plenitude:** For any consistent axiomatic theory T, there exists a structure S that satisfies T. Not merely in a model-theoretic "as if" sense, but as an ontic commitment: S exists.
2. **Anti-uniqueness:** Mathematics does not hinge on a single, correct universe (e.g., one V of set theory). Multiple mutually incompatible but consistent frameworks exist side-by-side.
3. **Benacerraf escape route:** Epistemic access does not require causal contact; mathematical knowledge is explained by our capacity to *refer to and reason about* any structure fixed by its axioms. On FBP, reference is easy because there is no scarcity problem: whichever structure your theory picks out (by its inferential role), that structure exists.

Bridge to mathematical monism.
FBP by itself is an **existence** thesis, not yet an **identity** thesis about physics. To reach monism, add a single step:

- **Identity step:** The physically actual world **is** (identical to) *one* member S* of the plenitude. Then "physical fact" = "fact about S*," and non-mathematical substrate

becomes explanatorily idle. FBP removes any ontic barrier to this identity: if all consistent structures exist, nothing forbids S* from being our world.

Explanatory advantages (once the identity step is added).

- **No-miracles redux:** Mathematics is effective because reality *is a case* of mathematics (S*). FBP's plenitude makes this unsurprising.
- **Modal clarity:** Physical possibility = membership in the theory-relative neighborhood of S* (e.g., models sharing axioms/invariants with S*). Counterfactuals become comparisons across nearby structures.
- **Underdetermination domesticated:** Multiple structures can fit current evidence; that is already FBP's picture of mathematics. Science's job is to shrink the equivalence class by adding constraints (symmetry, locality, computability, MDL compression), selecting the simplest S* that sustains all data.

Comparison with neighboring views.

- **Tegmark MUH:** MUH says "the universe is a mathematical structure," often with Level-IV: *all* structures exist. FBP supplies a rigorous philosophical version of that plenitude. MUH = FBP + explicit identity with a physically instantiated S* (and often a measure problem).
- **Ontic Structural Realism (OSR):** OSR says "only structure is real," but usually anchored in physics. FBP is purer: structure is real *independently* of physics; OSR becomes MUH once S* is identified.

Objections and replies.

- **"Explosion of ontology":** If everything consistent exists, isn't this profligate?
  Reply: Descriptive complexity is not ontic cost under FBP; parsimony applies to *theories we use*, not to the size of the mathematical domain. The *explanatory* object we pick (S*) is chosen by minimal description length relative to data; plenitude itself is not an empirical posit but a background stance that removes artificial scarcity.
- **"Reference problem":** How do we refer to *the* S*?
  Reply: By unique invariants fixed through theory choice: symmetry groups, conservation laws, computability/complexity bounds, and compression. As constraints accumulate, they canonically delineate S* up to isomorphism.
- **"Why this structure rather than another?"**
  Reply: That is precisely the scientific task. FBP does not prejudge S*; it legitimizes the search and blocks metaphysical dead-ends that would forbid identity with mathematics.

Operational program (FBP → MM).

1. Adopt FBP (plenitude).
2. Treat physics as the problem of isolating an S* in the plenitude that best compresses the total evidence under symmetry/locality/MDL constraints.
3. Read empirical success as evidence that our constraints are converging on invariants of S*.
4. Eliminate non-mathematical posits that neither refine S* nor improve compression/prediction.

Why Balaguer matters for mathematical monism.
FBP removes the last principled objection—"maybe the structure we need just doesn't exist"—by guaranteeing existence for any consistent candidate. Once existence is cheap, the rational move is the identity thesis: pick S* and let "world = S*." In that setting, physics is not a story about stuff obeying mathematics; physics *is* the ongoing isolation of which mathematics we inhabit.

IX. Ladyman: Ontic Structural Realism — relations without relata

Thesis in this lineage.
Ontic Structural Realism (OSR), developed prominently by James Ladyman (with Steven French), holds that the world is fundamentally **structure**: networks of relations governed by symmetries and dynamical constraints. The familiar "objects" are not basic; at best they are nodes abstracted from patterns of relations, and at worst they are dispensable fictions. In short: **relations without primitive relata.** If what exists is structure, and structure is mathematical, OSR is a near-direct conduit to mathematical monism.

Motivation: the metaphysics of modern physics.

1. **Permutation invariance and indistinguishability.** In quantum theory, elementary "particles" of the same type are indiscernible; state spaces are symmetrized or antisymmetrized. Individual essences drop out; what remains are representation-theoretic facts (group structure, statistics).
2. **Gauge redundancy.** Many variables in our best theories are gauge: multiple mathematically distinct descriptions represent the same physical situation. The invariant content is structural (quotienting by gauge).

3. **General covariance.** In GR, diffeomorphism-related models represent the same world. Coordinates and manifold points have no primitive identity; only diffeomorphism-invariant structure carries content.

Core commitments of OSR.

- **Primacy of structure:** The physically real content of a theory is its structure up to isomorphism (or a more refined categorical equivalence).
- **Anti-haecceitism:** No thisness without structure. If two alleged "worlds" agree on all structural facts, there is no further fact of the matter.
- **Object-eliminativism (strong OSR) or object-deflation (weak OSR):** Either objects do not exist fundamentally, or they are mere nodes/places in a relational web defined entirely by position in structure.

From OSR to math-only reality.
OSR by itself allows someone to say "the physical domain instantiates structure." Mathematical monism strengthens this to identity: **the physical domain just is the structure.** Why the upgrade is natural in OSR:

- If all admissible empirical content is exhausted by structural invariants, then any extra "carrier stuff" adds no new predictions or reductions in description length.
- Under a Minimum Description Length (MDL) norm, theories that posit only structure dominate rivals that add idle substrata.
- Therefore collapse "instantiation" to "identity" without loss: world = isomorphism class of the structure singled out by our best theory.

Formal sketch (representation-independent).

- Let T be a theory with model class M(T) and equivalence relation ~ expressing gauge/symmetry.
- Physical situations correspond to equivalence classes [X] in M(T)/~, and observables are invariants O([X]).
- If two presentations X and Y are isomorphic (X ~ Y), they represent the same world; the world is the class [X], not a presentation.
- Nothing in O([X]) requires a primitive inventory of objects; O depends only on morphisms, automorphism groups, and invariant quantities.

Explanatory dividends.

- **Unification:** Laws become constraints on allowable morphisms and invariants; objects and forces are not separate ontic kinds.
- **Modal clarity:** Possibility = membership in M(T)/~; counterfactuals are moves to nearby classes satisfying added constraints.
- **Symmetry as ontology, not ornament:** Noether-type correspondences and representation theory express what there is, not merely how we describe it.

Common objections and replies.

- **O1: "Relations need relata."**
  Reply: In category-theoretic and algebraic formulations, one can take arrows, functors, or algebraic operations as primitive. "Relata" are definable placeholders (positions in a structure), not extra substances. The identity of "nodes" is exhausted by their role in the web.
- **O2: "Structure must be instantiated."**
  Reply: Instantiation talk adds nothing once all empirical content is the structure up to isomorphism. Identity is the parsimonious reading.
- **O3: "OSR cannot explain qualia or measurement."**
  Reply: Observers and measurements are substructures with information-theoretic roles (decision procedures, records, couplings). If a purported remainder neither alters invariants nor compresses data, it is explanatorily idle by OSR's own lights.
- **O4: "Multiple inequivalent structures fit the data (underdetermination)."**
  Reply: This is a feature of science. OSR disciplines choice via symmetry, locality, computability, and MDL. Trade up when a structure yields strictly stronger, confirmed predictions or greater compression.

Operational OSR (toward MM).

1. **Extract invariants:** From a candidate theory, identify gauge-invariant quantities, symmetry groups, conserved charges, and categorical equivalences.
2. **Quotient presentations:** Work on M(T)/~; ban coordinates and labels from the ontology unless they induce invariants.
3. **Compare by compression:** Prefer structures that minimize description length while preserving (or expanding) predictive reach.

4. **Internalize observers:** Model detectors, records, and agents as definable substructures; read outcomes as invariant features of [X].
5. **Upgrade to identity:** Once no non-structural posit earns its keep, adopt world = [X].

Relation to other figures.

- **Geroch:** Provides the GR exemplar of structure-only content; OSR generalizes the lesson.
- **Balaguer:** Supplies plenitude on the mathematical side; OSR supplies parsimony and identity on the physics side.
- **Wheeler/Zuse/Fredkin:** Give constructive templates (bits, automata, reversible dynamics) that instantiate OSR's "relations-first" stance.

Why Ladyman matters for mathematical monism.
OSR is the **philosophical ratchet**: it blocks reintroduction of object-like "stuff" that does no explanatory work. If reality is nothing over and above invariant structure, and invariant structure is mathematical, then the cleanest reading is **mathematics-only**. OSR leaves no legitimate foothold for surplus ontology; the world is a specific structural class, and explanation is the derivation of its invariants.

X. French: Structural continuity and the elimination of objects

Thesis in this lineage.
Steven French develops Ontic Structural Realism (OSR) with a distinctive emphasis on **structural continuity** across theory change and on the **elimination or deflation of objects**. Where classical metaphysics centers enduring "things," French argues that what carries explanatory and inferential weight—*and what survives scientific revolutions*—are **structures, morphisms, and invariants**, not object-kinds with haecceistic identity. If only structure does genuine work, then objects can be **dispensed with** or **relegated to heuristic roles**, tightening the path to mathematical monism.

1) Structural continuity through theory change.

- **Problem:** Kuhn-style revolutions seem to replace one ontology with another (ether → fields; Newtonian masses → relativistic stress–energy), threatening realism.
- **French's move:** Track cross-theory **mappings of structure** (e.g., category-theoretic functors, limits, approximations, representation-theoretic correspondences) rather than cross-theory identities of objects.

- **UpShot:** What is preserved is **structural scaffolding**—symmetry groups, conservation patterns, dynamical relations—providing a realist core that is independent of object-inventory. Realism about structure, not things.

2) Eliminating objects (or cutting them down to size).

- **Indistinguishability in quantum theory:** Symmetrization/antisymmetrization removes individual essences; "which electron?" has no determinate answer.
- **Permutation invariance:** Physical states live in quotient spaces under particle permutations; individuation by labels is **gauge**.
- **Moral:** If objects are not even **in principle** individuable beyond their position in a web of relations, they are not fundamental posits. Objects reduce to **structural roles** (places in a representation), not extra stuff.

3) Gauge, surplus structure, and invariants.

- **General covariance and gauge theories:** Multiple mathematically distinct presentations encode the same physical situation; only **gauge-invariant** content is physically meaningful.
- **French's emphasis:** Treat representational redundancy as a lesson: *do not reify what disappears under quotienting.* The ontology is the **equivalence class** (the isomorphism/invariant content), not any decorated presentation.

4) Formal sketch: structural preservation over objects.

- Let T_1, T_2 be successive theories; let $\mathcal{M}(T_i)$ be their model categories with quotient by gauge/symmetry $\sim_i$.
- A **bridge** $F: \mathcal{M}(T_1)/\sim_1 \to \mathcal{M}(T_2)/\sim_2$ preserves selected invariants (e.g., conserved quantities, symmetry actions, limiting dynamics).
- **Continuity criterion:** Real content = invariants preserved (up to isomorphism) by such bridges.
- **Elimination criterion:** Posits not preserved or not invariant are **representation artifacts**; exclude them from ontology.

5) Explanation without objects.

- **Why does X occur?** Because the structure has property P (e.g., Noether correspondence under a symmetry group G; stability from eigenstructure; universality from renormalization-group fixed points).

- **No need** for "hidden carriers." The explanatory arrows point to **structural facts** (group actions, functorial relationships, spectral features), not to substrata.

6) Structural tools French champions.

- **Group theory & representation theory:** individuate physical kinds via irreps, not "thisness."
- **Category theory:** articulate theory change and inter-theory relations as functors/natural transformations; emphasize morphisms over elements.
- **Algebraic formalisms:** focus on algebras of observables, states as functionals, and invariant subalgebras under symmetry, minimizing commitment to "points" or "particles."

7) Objections and replies.

- **O1: Relations require relata.**
  Reply: In many mathematical settings, **arrows are primitive**; "objects" are nodes *only* insofar as they sustain morphisms. Identity is role-relative (up to isomorphism). No extra, haecceistic posit is needed.
- **O2: Loss of explanatory grip (no things to hold onto).**
  Reply: Grip is gained, not lost: explanation becomes **proof of invariants** and **derivation of patterns**; predictive success attaches to structural machinery, which is exactly what transfers across domains.
- **O3: Multiple structures fit the data (underdetermination).**
  Reply: Science already resolves ties using **symmetry, locality, computability/complexity, and MDL**. Prefer the structure with maximal invariance and minimal description length; trade up when evidence discriminates.

8) Relation to neighboring positions.

- **Versus Geroch:** Geroch exhibits structure-only content within GR; French generalizes the lesson across physics and centers **theory change**.
- **With Ladyman:** Jointly articulate OSR; French often presses the **eliminativist** edge on objects and the **methodological priority** of invariants.
- **Toward monism:** Once objects are deflated and only structural invariants remain as carriers of content, the simplest move is identity: **world = mathematical structure** singled out by our best, most invariant, most compressed theory.

9) Operational discipline (French-to-MM).

1. **Work modulo symmetry/gauge:** formulate models in quotient spaces from the outset.
2. **Extract invariants:** conserved quantities, symmetry orbits, topological indices, spectral gaps.
3. **Bridge theories structurally:** build and test functorial maps across regimes (limits, approximations, dualities).
4. **Score by compression:** choose structures that minimize description length while maximizing preserved invariants and empirical reach.
5. **Drop surplus ontology:** if a posit does not survive quotienting or increase compression/prediction, eliminate it.

Why French matters for mathematical monism.
French tightens the vise: by making **structural continuity** the currency of realism and by **eliminating objects** that can't be individuated beyond role-in-structure, he leaves **nothing but mathematics** to do the ontological lifting. If explanation = invariant structure, continuity = structural mapping, and objects = dispensable labels, then the cleanest reading is the monist identity: **reality just is the mathematical structure encoded by our most invariant, most compressive theory.**

XI. Marchal: Arithmetical realism; observers inside arithmetic

Thesis in this lineage.
Bruno Marchal advances **computationalism + arithmetical realism**: (i) conscious states supervene on computations implementable by a universal machine; (ii) the truths of arithmetic (e.g., PA or stronger) are mind- and world-independent. From these, he argues that **physics is an appearance generated from inside arithmetic**: the "laws of physics" are statistical regularities over the space of computations arithmetically defined. In short: if comp is true, the right ontology is arithmetic; what we call "matter" is an internal, first-person measure on number-theoretic computation histories.

Key pillars.

1. **Computational supervenience (comp).**
   Replace brains by functionally equivalent computations without loss of first-person experience. Identity tracks the computation, not the substrate. By Church–Turing, this reduces experience to computations realizable by a universal Turing machine (UTM).

2. **Arithmetical realism.**
   Arithmetic truths exist independently of minds and worlds. Computations are definable subsets of N with purely arithmetical predicates (e.g., "there exists a proof in PA that UTM halts on input x in t steps with trace y").

3. **Universal Dovetailer (UD).**
   Consider a dovetailing program that enumerates **all** programs on a UTM and interleaves one step from each (UD* includes all infinite continuations). This generates the ensemble of all computable observer-moments somewhere in the space of runs.

4. **First-person indeterminacy.**
   If personal identity rides on computational continuation, and the UD spawns all extensions consistent with one's current computational state, then the next experienced state is **not unique**. It is sampled from a measure over all consistent continuations (self-locating uncertainty).

5. **Measure problem turned physics.**
   The "physical laws" we observe are precisely the **regularities typical** under the first-person measure induced by the UD on consistent continuations. Physics = statistics of arithmetical computations seen from the inside.

Core arguments (sketch).

- **UDA (Universal Dovetailer Argument).**
  Step A: Assume comp. Step B: Teleportation/substitution shows substrate independence. Step C: Indeterministic reconstitution (multiple copies) yields self-locating uncertainty. Step D: UD generates all such branches arithmetically.
  Conclusion: First-person predictions must be probabilities over arithmetically defined computation-continuations; any "primitive matter" is explanatorily idle.

- **MGA (Movie-Graph Argument).**
  Attempts to pin consciousness to a concrete "movie" of brain states lead to absurdities: replaying frames on inert media should not sustain consciousness, yet functional equivalence pressures say it would unless we relocate ontology to the **abstract computation**. This pushes identity to arithmetic description, not physical substrate.

- *Modal logics of self-reference (G, G).**
  Let a sound arithmetical machine reason about itself using provability/consistency (GL, arithmetized).

    - G: what the machine can prove about itself (provability modality).

- G*: what is actually true about the machine's proofs (truth about provability). Marchal extracts distinct "first-person" and "third-person" logics: the inner experiential perspective (with uncertainty) versus the outer God's-eye proof theory. Physics should align with the **first-person** modal theory induced by arithmetic.

From arithmetic to appearances.

- **Ontology.** Arithmetic only (no primitive matter). Programs = numbers; runs = arithmetically definable relations on numbers.
- **Observers.** Definable clusters of computation states with appropriate counterfactual structure (robustness under small code/context variations).
- **World-regularities.** Laws are theorems about typicality under the first-person measure on UD* continuations that preserve one's computational invariants.
- **Prediction.** To predict an outcome, compute the relative measure of continuations in which that outcome occurs; Born-like weights should, in principle, emerge from counting/measure on branches respecting computational symmetries (Marchal sketches routes to quantum amplitudes via measure-on-histories constraints).

Formal taste (copy-safe).

- Let U be a fixed UTM; let Prog be the set of all finite programs p (natural numbers).
- UD interleaves U(p) one step at a time; UD* = set of all infinite traces tau = (config_0, config_1, …) consistent with U's transition function.
- Fix an observer-moment description s (a finite computational state pattern). Define Ext(s) = { tau in UD* : tau contains s and extends it coherently }.
- Define a first-person measure m_s over Ext(s) by an arithmetically specifiable weighting (e.g., prefix-free algorithmic probability over minimal programs realizing each extension, corrected for dovetailing multiplicity and counterfactual robustness).
- Prediction = argmax over outcomes O of m_s({tau in Ext(s): O holds in the next k steps}).

Why this serves mathematical monism.

- Every entity in the construction is arithmetical: codes, machines, proofs, traces, measures.
- "Matter" is redundant: adding a primitive substrate does not change m_s nor refine predictions.

- By MDL/Occam, world = arithmetic; "physics" = first-person statistics over UD*.

Comparisons.

- **Tegmark MUH:** "World is a mathematical structure." Marchal: "World-as-experienced is a probability distribution over arithmetical computations"; physics is derived, not posited.
- **OSR (Ladyman/French):** "Only structure is real." Marchal specifies the structure and the observer's place within it (self-reference logics + UD measure).
- **Zuse/Fredkin:** computation is fundamental; Marchal relocates even the machine to arithmetic itself (no hardware remainder).

Objections and replies.

- **Measure non-uniqueness.** Which weighting over UD*?
  Reply: Constrain by invariance (prefix-free coding), counterfactual robustness, and reflection principles from the machine's self-reference logic. Competing measures are tested by empirical adequacy and compressive power.
- **Quantum amplitude recovery.** How do complex amplitudes arise from counting?
  Reply: Marchal sketches derivations tying first-person indeterminacy to interference-like constraints; full derivation remains programmatic, but any rival primitive-matter posit must yield strictly better compression/prediction.
- **Indexical vagueness (who is "I"?).**
  Reply: "I" = an equivalence class of computational states up to functional indistinguishability that sustains coherent counterfactuals. This is standard in comp philosophy of mind; arithmetic makes it precise.
- **Arithmetical realism burden.**
  Reply: If arithmetic is denied, we lose the very machinery used everywhere in physics. Accepting its objectivity is the minimal commitment; comp then moves the rest.

Why Marchal matters for mathematical monism.
He supplies an explicit **derivational route**: assume comp and arithmetical realism, and the empirical face of "physics" emerges as a **first-person probability calculus over arithmetically defined computations**. No non-mathematical remainder does explanatory work. On this view, to exist is to be arithmetically capturable; to observe is to sample a computable continuation; to explain is to prove a typicality theorem inside arithmetic. That is monism, executed.

XII. Schmidhuber: Algorithmic TOE and the ensemble of programs

Thesis in this lineage.
Jürgen Schmidhuber develops an Algorithmic Theory of Everything (AToE) in which **all computable histories exist**: every finite program p on a fixed universal computer U is run (e.g., by dovetailing), and its infinite output stream defines a universe/history. The **a priori measure** over histories is given not by metaphysical fiat but by **algorithmic probability**: simpler programs (shorter descriptions) get exponentially higher prior weight. Physics becomes **Solomonoff-style induction** over the ensemble; the "laws" we infer are the shortest programs that best compress our observation sequence. If the world is one such computable history—and nothing over and above it—then the world **is** mathematics.

Core ingredients.

1. **Programs generate universes.**
   Fix a prefix universal Turing machine U. For any program p, let U(p) be an infinite output stream (or a stream that halts, then pads). Read each stream as a candidate "world history" h. The **ensemble** E = { U(p) : p ranges over all finite binary strings } is the totality of computable histories.

2. **Algorithmic probability (Solomonoff-Levin).**
   The universal semimeasure over strings x is
   M(x) = sum_{p: U(p) starts with x} 2^{-len(p)}.
   Short programs dominate: the prior probability of observing an initial segment x is governed by how **compressible** x is. Prediction proceeds by **universal induction**: choose continuations y that maximize M(xy).

3. **Occam's razor as mathematics.**
   Kolmogorov complexity K(x) is the length of the shortest program producing x. AToE asserts that, given long enough observation prefixes, the best predictive "law of nature" is the shortest program consistent with those data (Minimum Description Length). "Law" = compressed description of the data stream.

4. **Speed prior and computability.**
   Because M uses an incomputable sum, Schmidhuber proposes **speed-sensitive** computable priors (e.g., the Speed Prior S) that downweight histories requiring excessive computation. Result: a bias toward worlds that are not only **simple** but also **fast to compute**.

5. **Levin search and asymptotic optimality.**
   Universal search (Levin) interleaves all candidate programs with a provable bound: it is within a constant factor of the fastest possible solver, up to multiplicative overhead. In

this picture, scientific discovery is Levin-like: simulate many simple candidate laws and keep the one with best compression/prediction tradeoff.

6. **Self-improving agents (Gödel machines).**
   Extend the ensemble idea to agents: a Gödel machine rewrites its own code once it can **prove** that the rewrite improves expected utility. Even agency becomes a mathematical object in the same formal universe of programs.

From ensemble to monism (identity step).
AToE begins with an **existence claim**: all computable histories exist. The **monist** move is to identify our world with **one** history h* in E. Then:

- Objects, fields, and laws are **patterns** in h*.
- Modal talk ("could have been otherwise") = neighboring computable histories h with shorter/longer descriptions consistent with earlier prefixes.
- Explanation = compression: show that a short program p* produces (and predicts) the observed prefixes of h*.
- No extra, non-mathematical "substance" increases predictive power or decreases description length. Hence, by MDL/Occam, **world = mathematics (the program and its run).**

Contrasts and continuities.

- **Zuse/Fredkin:** one specific CA or reversible lattice gas as the world; Schmidhuber widens to **all** programs, then uses priors to argue we find ourselves in a highly compressible, efficiently computable one.
- **Marchal:** UD over computations with first-person measure; Schmidhuber: universal prior over programs with **third-person** predictive semantics. Both rely on dovetailing; they differ in how they assign **measures** and where "observation" enters.
- **Tegmark (MUH):** all mathematical structures exist; AToE narrows to **computable** ones and equips them with a **measure** grounded in algorithmic information.

Explanatory payoffs.

1. **No-miracles redux (formalized).** Mathematics is effective because the universe we inhabit is **itself** a compact piece of code; elegant theories win because priors reward short descriptions.

2. **Predictive discipline.** Given an observation prefix x, universal induction says predict y that maximizes M(xy). This yields a concrete, if incomputable, **gold standard** of prediction; computable speed-priors approximate it.

3. **Occam as physics.** The shift from "stuff governed by laws" to "data compressed by law-code" collapses ontology and methodology: physics is the search for the shortest predictive program p* for our data stream.

4. **Anthropic flavor without mystique.** Observers typically appear in those histories with structures capable of supporting them; algorithmic probability favors histories where such complexity **arises cheaply** from simple code, not from gigantically contrived initial conditions.

Operational program (AToE-to-MM).

1. **Fix U and prior.** Choose a reference UTM (prefix), and a computable prior approximating M (e.g., speed prior S). Acknowledge constant-factor dependence on U as representational, not ontic.

2. **Compression as law discovery.** For empirical data D (cosmology, particle events), search for minimal programs that reproduce D and generalize to D'. Use MDL: score(p) = len(p) + len(residuals given p). Prefer lower score.

3. **Forecasting.** For candidate continuations, weight by prior mass assigned to programs that produce those continuations. Pick the continuation with highest posterior.

4. **Model selection across theories.** Competing "theories" are just program classes. Prefer those with better compression/prediction; trade up when evidence shifts the posterior.

5. **Retire surplus metaphysics.** If a non-mathematical posit cannot be cashed out as a reduction of description length or improvement in predictive accuracy, eliminate it.

Common objections and replies.

- **Incomputability of M.**
  Reply: Use computable approximations (speed prior); the conceptual point stands: parsimony is mathematically enforced.

- **Reference machine dependence.**
  Reply: K(x) is machine-dependent up to an additive constant; for long data and theory comparison, constants wash out. Scientific practice already tolerates such gauge.

- **Quantum amplitudes and continuity.**
  Reply: AToE is compatible with computable quantum models (quantum cellular

automata, computable reals approximations). If nature used uncomputable reals, no finite experiment could discriminate them from computable surrogates of sufficient precision; priors still reward simpler surrogates.

- **Measure problem (which worlds are "typical"?).**
  Reply: Algorithmic priors resolve it by fiat of mathematics: typical = high prior mass, i.e., histories with short, fast programs. This is not arbitrary; it is the unique universal dominance property of Solomonoff-Levin style priors.

Why Schmidhuber matters for mathematical monism.
He provides a **fully formal** route from data to ontology: (i) everything real is generated by programs; (ii) predictive success is compression; (iii) priors mathematically privilege simple, fast code; (iv) nothing beyond the program and its run adds empirical bite. On this view, to exist is to be computably generable; to explain is to shorten description length; to predict is to perform universal induction. Once accepted, the identity move is irresistible: *the world is the mathematical object (U, p, run)** that best compresses the total evidence.

XIII. Tegmark: MUH and Level-IV reality as mathematical structure

Thesis in this lineage.
Max Tegmark's Mathematical Universe Hypothesis (MUH) states the identity claim in its bluntest form: **physical reality = a mathematical structure**. Not "instantiated by," "described by," or "isomorphic to," but **identical with**. His further **Level-IV multiverse** asserts **ontic plenitude**: *all* consistent mathematical structures exist "out there." MUH is thus mathematical monism plus maximal existence.

Core claims.

1. **External existence = mathematical existence.**
   The only attributes that survive under idealization are structural (axioms, objects, relations). If everything empirically accessible is structural, then positing a non-mathematical "carrier" is idle; identify world with structure.

2. **No baggage (observer-free formulation).**
   A legitimate candidate for reality must be specifiable without indexicals or anthropic tags—just a bare structure S given by axioms or constructive rules.

3. **Ergodic epistemology.**
   The regularities we observe are theorems about S. Different "laws" across regions correspond to different substructures or effective descriptions within S.

Level-IV multiverse (plenitude).
Where Levels I–III vary initial conditions, constants, or quantum branches inside one background framework, Level IV **changes the underlying mathematics itself**: every consistent structure S exists. This universalizes explanation: if an observed regularity holds, it is because we inhabit a structure in which that regularity is a theorem. Apparent fine-tuning becomes **typicality** relative to the ensemble of structures that support observers.

Identity step and minimality.

- Suppose our best theory T singles out a class of models M(T) modulo equivalence ~ (gauge/symmetry).
- If no further non-structural posit yields extra predictive bite or reduces description length, then world = [S*] for some S* in M(T)/~.
- Level IV removes the residual metaphysical worry ("does S* really exist?"): by stipulation, **every** consistent S does.

Formal sketch (copy-safe).

- A mathematical structure is a tuple S = (D; R_1, R_2, ..., F_1, F_2, ...), with domain D, relations R_i, and functions F_j satisfying axioms A(S).
- MUH: There exists S* such that **world = S***. All empirical statements reduce to theorems about S*.
- Level IV: For every consistent set of axioms A, there exists S_A. The "cosmos" is the class { S_A : Con(A) }.

Explanatory payoffs.

1. **No-miracles (completed).** The "unreasonable effectiveness of mathematics" is no miracle if reality is mathematics. Derivations in physics are literally theorems about S*.
2. **Modal clarity.** Physical possibility = model-theoretic possibility inside S*; metaphysical possibility = consistency across the Level-IV class. Counterfactuals are moves within M(T) or across structures that share relevant subtheories.
3. **Parsimony and MDL.** If empirical content is wholly structural, MUH eliminates dual bookkeeping (laws + stuff). Minimal Description Length (MDL) then favors the shortest axiomatization or program generating S*'s empirical boundary conditions.
4. **Unification of "why these laws?"** Because we are in an S* where those laws are theorems and where observer-supporting substructures are typical enough under the relevant measures.

Objections and replies.

- **O1: Measure problem on Level IV.**
  Objection: If all structures exist, how do we assign weights/typicality?
  Reply: MUH per se is agnostic about a unique measure; several routes are live: (i) algorithmic priors (Schmidhuber-style); (ii) anthropic filters constrained by invariance; (iii) internal symmetry/MDL criteria that privilege simpler S*. The empirical standard is *comparative*: adopt the weighting that best compresses and predicts observations; refine as needed.

- **O2: Semantic emptiness: "math exists abstractly; physics is concrete."**
  Reply: The abstract/concrete contrast does no explanatory work once all observables are structural invariants. "Concrete" becomes a role word for certain substructures (e.g., those supporting observers) within S*.

- **O3: Underdetermination: many S fit current data.**
  Reply: True—and routine in science. MUH adds no extra underdetermination; it clarifies the choice rule: prefer structures that maximize invariance and minimize description length while matching data. Trade up when evidence discriminates.

- **O4: Incompleteness/consistency worries.**
  Reply: MUH quantifies over *consistent* structures. Incompleteness means that some truths of S* are undecidable from A(S*), not that S* fails to exist. Physics already tolerates undecidable frontiers; MUH reinterprets them as genuine features of S*.

- **O5: "But qualia."**
  Reply: Observers are definable substructures (information-processing systems) satisfying decision/memory constraints. If adding non-mathematical "feel" neither changes predictions nor reduces description length, it is explanatorily idle under MUH's criterion of adequacy.

Operational MUH (how to use it).

1. **Axiomatics/programs, not prose.** Formulate candidate S via axioms or generative code. Ban anthropic or coordinate baggage from the core definition.

2. **Equivalence first.** Work modulo gauge/symmetry; treat presentations that are isomorphic (or more generally categorically equivalent) as the same physical situation.

3. **Compression as progress.** Given total data D, select S (or effective theory T_S) minimizing MDL: score(S) = len(axioms or code of S) + len(D | S). Prefer structures with richer symmetry subject to fit.

4. **Modal hygiene.** Handle counterfactuals inside S (model-class moves) before invoking Level IV; use Level IV only when explanation requires cross-structural comparison (e.g., fine-tuning narratives).
5. **Anthropic discipline.** When invoking observer conditions, formalize them as structural constraints (computational capacity, thermodynamic openness, causal graph features) to avoid ad hoc selection.

Relations to the lineage.

- **Pythagoras/Plato:** From "all is number" and "only the mathematical is knowable" to **identity**.
- **Wheeler/Zuse/Fredkin:** Provide concrete constructive templates; MUH abstracts the lesson: the template is not privileged—*any* consistent S can be real.
- **Geroch/Ladyman/French:** Their structural eliminativism about objects/coordinates is the philosophical scaffolding MUH uses to go from "instantiated structure" to "structure, full stop."
- **Balaguer:** FBP's plenitude is the philosophical cousin of Level IV; MUH adds the physical identity claim.

Why Tegmark matters for mathematical monism.
MUH is the **closure principle** of the program. Earlier sections progressively strip away surplus ontology until only structure does explanatory work. Tegmark codifies the endpoint: there is no gap to bridge—**what exists is a mathematical structure**; the apparent variety of "universes" is just the variety of consistent structures. The scientific task is then crisp: infer which S* we inhabit by maximizing invariance and minimizing description length over total evidence, and treat all further metaphysics as surcharge unless it buys new, testable compression.

XIV. Conclusion: From description to identity — closing the explanatory gap

We began with a caution: "mathematics describes" does not yet mean "mathematics is." The historical arc assembled here shows how repeated attempts to keep a gap between description and reality collapse under their own success. Pythagoras moves from consonance to ontology; Plato codifies the epistemic filter: only the mathematical is knowable. Wheeler relocates being to recorded distinctions (it-from-bit). Zuse exhibits a fully explicit identity model (world as cellular automaton). Fredkin tightens the screw with reversibility and information-conservation, making dynamics a permutation on a discrete state space. Geroch and OSR (Ladyman, French) strip away surplus ontology, leaving only equivalence classes of structures and their invariants.

Balaguer guarantees plenitude so that the "structure we need" exists. Marchal derives physics as first-person statistics over arithmetical computations. Schmidhuber formalizes Occam as a prior over programs, converting "law" into optimal compression. Tegmark states the endpoint without euphemism: world = mathematical structure; Level IV = all consistent structures.

Across these stations, three criteria recur:

1. Invariance: retain only what survives symmetry/gauge; treat isomorphic presentations as one.
2. Sufficiency: if structure alone yields the total empirical content, extra "stuff" is idle.
3. Compression (MDL): prefer theories that minimize code length for laws + data.

Read together, these make "identity" the rational upgrade from "description." If the predictive and explanatory work is wholly done by structure, if counterfactuals and modalities are internal to model classes, and if every non-structural posit either duplicates content or lengthens the shortest code, then the map–territory distinction is bookkeeping, not metaphysics.

Objections remain: measure over plenitude, reference to a unique S*, the fate of qualia, computability vs. continuity. But each objection is either (i) a technical program inside mathematics (measures, selection principles, approximation theory), or (ii) a demand for an extra posit that neither sharpens predictions nor shortens description length. Under comparative adequacy, mathematics keeps winning.

Hence the closing stance:

- Ontology: the world is a mathematical structure S*.
- Epistemology: knowing the world is inferring S* (up to equivalence) by maximizing invariance and minimizing description length over total evidence.
- Explanation: to explain is to prove an invariant or exhibit a compression.
- Modality: possibility = membership in the relevant model class; counterfactuals = structured moves within or across nearby S.
- Anthropic discipline: observer talk reduces to structural constraints (information processing, thermodynamic openness, causal graph properties).

What began as an aesthetic hunch—"all is number"—matures into a methodological ratchet: at each turn, mathematics absorbs the explanatory load, and nothing external earns its keep. The gap between description and reality closes, not by decree, but by the steady elimination of idle posits. On this view, the most sober thesis left standing is also the starkest: the only reality we know—and need—is mathematics.

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